

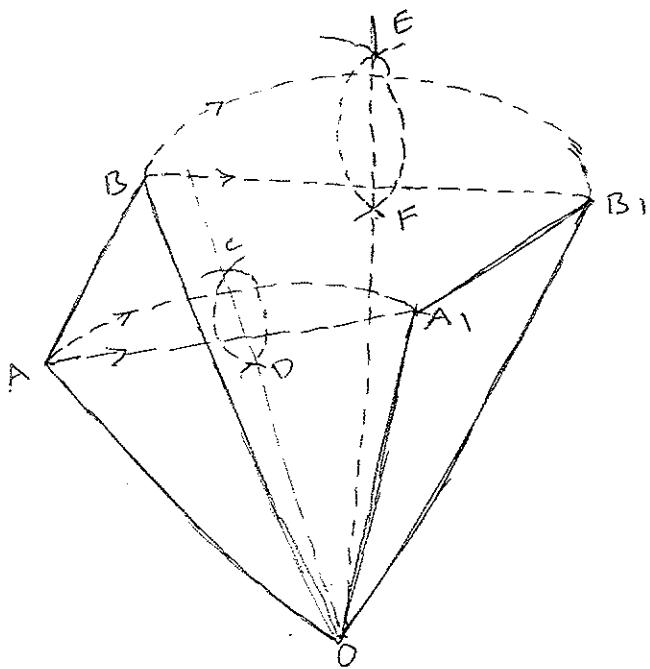
PLANE MOTION OF BODY*) Instantaneous centre :-

The instantaneous centre of a moving body may be defined as that centre which goes on changing one instant to another.

The locus of all such instantaneous centres is known as centroid. A line drawn through an instantaneous centre and perpendicular to the plane of motion is called instantaneous axis and locus of this axis is known as anode.

*) Instantaneous centre Method OF Rotation :-

This method is convenient and easy to apply in simple mechanisms.



Let the link AB in a short interval of time changes its position from AB to A_1B_1 . The point A of the link moved to A_1 , where the point B of the link has moved to the point B_1 , as shown in Fig.

Draw the right bisectors of chord AA_1 and chord BB_1 .

Let 'CD' is the right bisectors of AA_1 , where as 'EF' is the right bisector of BB_1 . Let these two bisectors meet at the point 'O'. Then the point 'O' is the instantaneous centre of rotation of the link AB. This means the link AB as a whole has rotated about 'O'.

Let V_A = linear velocity of point A

V_B = linear velocity of point B

ω = angular velocity of link AB about 'O'.

This means the angular velocity of point A and point B about O will also ω .

we know that,

$$\text{linear velocity} = \text{Angular velocity} * r$$

\therefore linear velocity of point A is given by

$$V_A = \text{Angular velocity of point A about } O * \text{distance of } A \text{ from centre of rotation}$$

$$V_A = \omega * AO$$

$$\omega = \frac{V_A}{AO} \quad \text{--- --- (i)}$$

Similarly, the linear velocity of point B is given by

$V_B = \text{Angular velocity of point B about O} \times \text{Distance of B from centre of rotation (i.e. from O)}$

$$V_B = \omega \times BO$$

$$\omega = \frac{V_B}{BO} \quad \text{--- --- (ii)}$$

Equating eqn (i) and (ii), we get

$$\frac{V_A}{AO} = \frac{V_B}{BO} = \omega$$

(or)

$$\frac{V_A}{V_B} = \frac{AO}{BO}$$

The direction of the velocity at A will be at right angle to AO
whereas the velocity at B will be at right angle to BO.

If 'C' is any other point on the link AB, then the velocity of C, i.e. V_C can be determined by

$$\therefore \frac{V_A}{AO} = \frac{V_B}{BO} = \frac{V_C}{CO} = \omega$$

where $CO = \text{distance of C from instantaneous centre 'O'}$.

* Number and types of Instantaneous centre

The number of instantaneous centre in a constrained kinematic chain is equal to the number of possible combination of two links. The number of pair of links or the number of instantaneous centre is the number of combination of 'n' links taken two at a time. Mathematically, number of instantaneous centre.

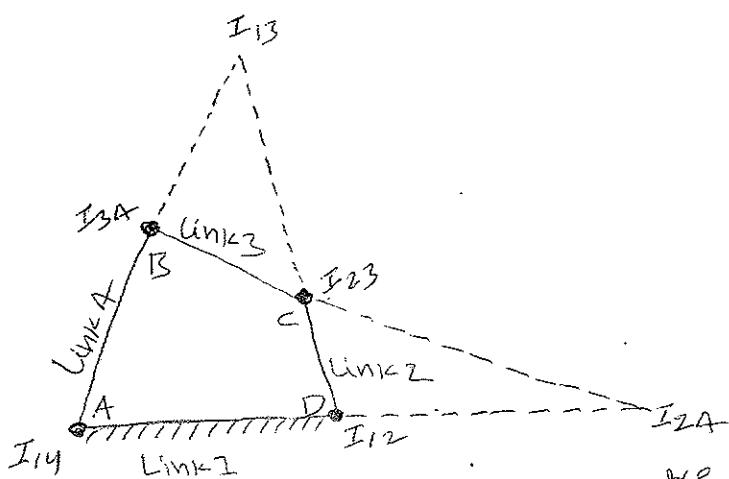
$$N = \frac{n(n-1)}{2}, \text{ where } n = \text{number of links.}$$

Types :- (1) Fixed instantaneous centre

(2) Permanent instantaneous centre

(3) neither fixed nor permanent instantaneous centre.

The first two types i.e. fixed and permanent instantaneous centres are together known as primary instantaneous centres and third type is known as secondary instantaneous centres.



* Fig:- Types of Instantaneous centre

Consider a Four bar mechanism ABCD as shown in Fig. The number of instantaneous centres [N] in a four bar mechanism is given by

$$N = \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 6$$

The instantaneous centre I_{12} and I_{14} are called the Fixed instantaneous centres as they remain in the same place for all configurations of the mechanism.

The instantaneous centres I_{23} and I_{34} are of permanent nature instantaneous centres as they move when the mechanism moves, but the joints are permanent in nature.

The instantaneous centres I_{13} and I_{24} are neither fixed nor permanent instantaneous centres as they vary with configurations of the mechanism.

Kennedy Theorem (or Three centre-in-line Theorem) :-

It states that if three kinematic link move relatively to each other, their instantaneous centres [three in number i.e $\frac{3*(3-1)}{2} = 3$] will lie on a straight line.

Procedure for locating instantaneous centre:-

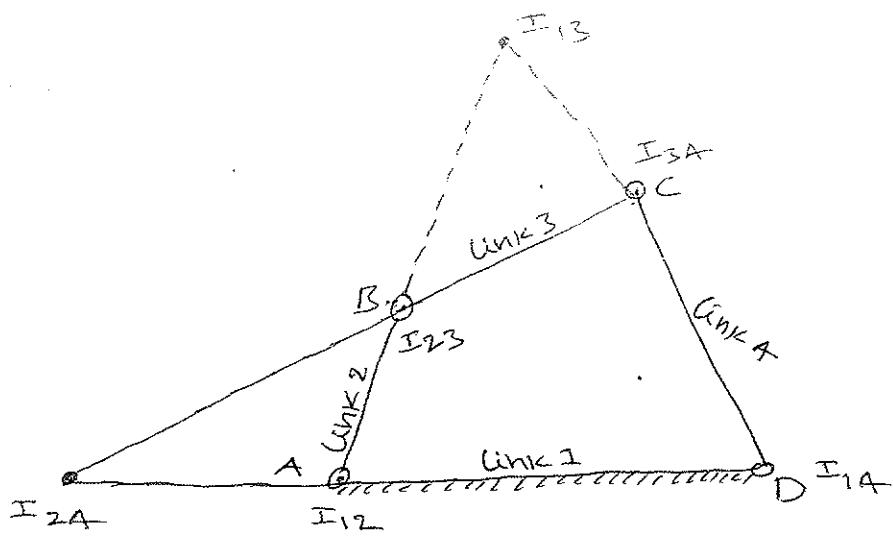
The following figure shows a Four bar mechanism ABCD. The procedure of locating instantaneous centre is given

- (1) determine the number of instantaneous centre [N] by using the relation

$$N = \frac{n(n-1)}{2}$$

where $n = \text{number of links} = 4$ (here)

$$N = \frac{4(4-1)}{2} = 6.$$



② Prepare a list of all the instantaneous centres in a tabular form

Link	1	2	3	4
Instantaneous centres	I ₁₂	I ₂₃	I ₃₄	-
	I ₁₃			
	I ₁₄			

- 3) By inspection, locate Fixed and permanent instantaneous centres. For the Four bar mechanisms shown in Fig. these are I₁₂, I₁₄, I₂₃, and I₃₄. The Instantaneous centre I₁₂ and I₁₄ are fixed whereas I₂₃ and I₃₄ are permanent.
- 4) Locate the remaining neither fixed nor permanent instantaneous centre by Kennedy's theorem.

Consider three links 2, 3 and 4. The instantaneous centre for links 2 and 3 is I₂₃ and for 3 and 4 is I₃₄. The instantaneous centre for links 2 and 4 (i.e. I₂₄) according to Kennedy's theorem should be ^{in a} line of I₂₃ and I₃₄. Hence draw a straight line passing through I₂₃ and I₃₄ and produce if necessary.

Now consider three links 1, 2, and 4. The instantaneous centre for links 1 and 2 is I₁₂ whereas for links 1 and 4 is I₁₄. The instantaneous centre for links 2 and 4 (i.e. I₂₄)

According to Kennedy's theorem should be in line of I_{12} and I_{14} .
Hence draw a straight line passing through I_{12} and I_{14} and produce.
This line intersects the line already drawn through I_{23} and I_{34} at
 I_{24} . Thus the Instantaneous centre I_{24} is located.

⑤ similarly the instantaneous centre I_{13} is located by considering three
links 1, 2 and 3. The I-centre for links 1 and 2 is at I_{12} whereas
the instantaneous centre for links 2 and 3 is at I_{23} . Draw a straight
line passing through I_{12} and I_{23} and produce this line. According to
Kennedy's theorem, the I-centre for links 1 and 3 must lie on this
straight line.

Now consider three links 1, 2 and 3. The I-centre for
links 1 and 2 is at I_{12} whereas for links 2 and 3 is at I_{23} .
The I-centre for links 1 and 3 must lie on a straight line
passing through I_{12} and I_{23} . Draw the straight line passing
through I_{12} and I_{23} . Produce this line which intersects the
straight line drawn through I_{14} and I_{34} at I_{13} . Thus the
I-centre I_{13} is located.



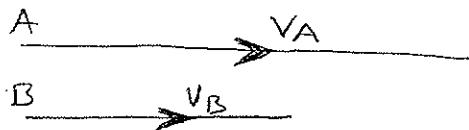
Relative velocity method

The relative velocity method for determining the velocities of different points in the mechanism.

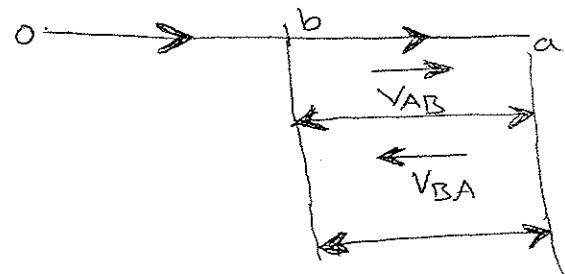
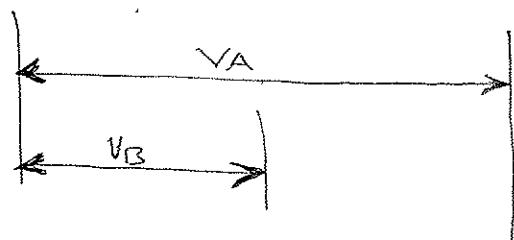
* Relative velocity of two bodies moving in straight lines :-

The relative velocity of two bodies moving along (i) parallel lines
 (ii) inclined lines

as shown in Fig:



Fig(a):



Fig(b):

Fig:- Relative velocity of two bodies moving along parallel lines.

Consider two bodies A and B moving along parallel lines in the same direction with absolute velocities v_A and v_B such that $v_A > v_B$, as shown in Fig(a). The relative velocity of A with respect to B:

$$v_{AB} = \text{vector difference of } v_A \text{ and } v_B = \overline{v_A} - \overline{v_B}$$

From Fig(b). The relative velocity of A with respect to B (i.e v_{AB}) may be written in the vector form as follows:

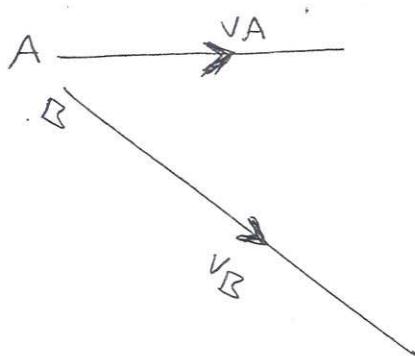
$$\boxed{\overline{v_{ba}} = \overline{va} - \overline{vb}}$$

Similarly, the relative velocity of 'B' with respect to A,

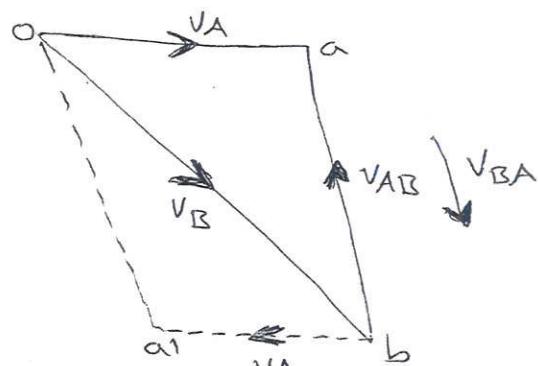
$$v_{BA} = \text{vector difference of } v_B \text{ and } v_A = \overline{v_B} - \overline{v_A}$$

$$\therefore \overline{ab} = \overline{ob} - \overline{oa}$$

ii Inclined line :-



Fig(a).



Fig(b)

Fig(2) :- Relative velocity of two bodies moving along inclined line.

Now consider the body B moving in an inclined direction of shown in Fig (a). The relative velocity of A with respect to B may be obtained by the law of parallelogram of velocities (or) triangle law of velocity.

~~Take~~ Take any fixed point 'O' and draw vector oa to represent v_A in magnitude and direction to some suitable scale. Similarly, draw the vector ob to represent v_B in magnitude and direction to the same scale.

The vector \bar{ba} represents the relative velocity of A with respect to B as shown in Fig. In the similar way as discussed above, the relative velocity of A with respect to B,

$$v_{AB} = \text{vector difference of } v_A \text{ and } v_B = \bar{v}_A - \bar{v}_B$$

$$\bar{ba} = \bar{oa} - \bar{ob}$$

Similarly, the relative velocity of B with respect to A,

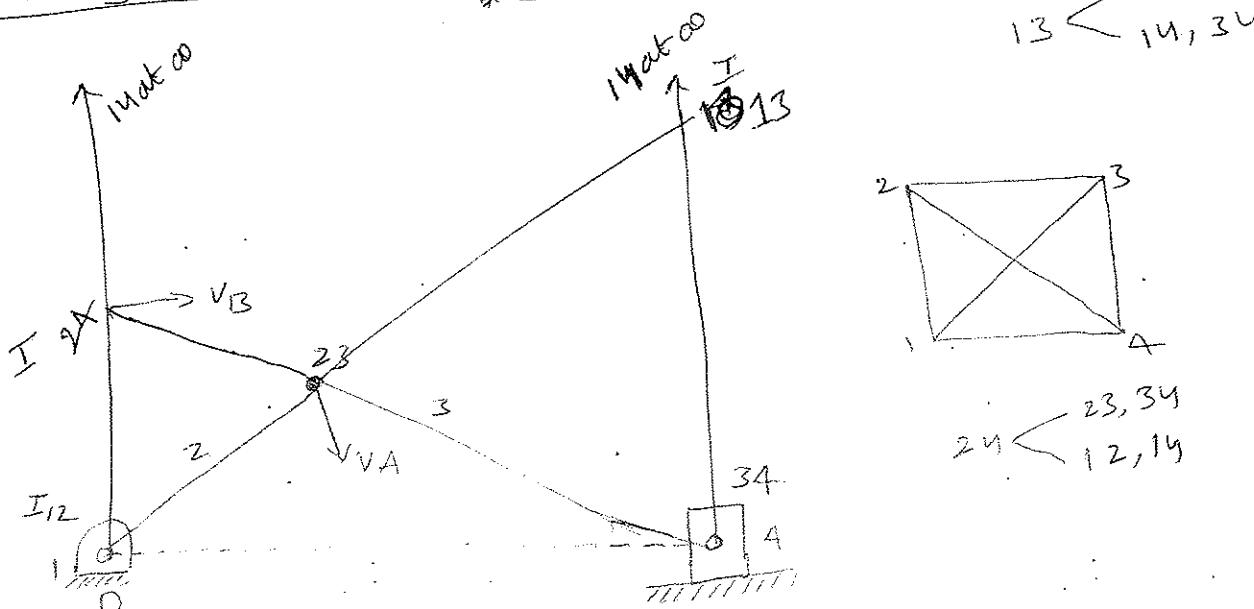
$$v_{BA} = \text{vector difference of } v_B \text{ and } v_A = \bar{v}_B - \bar{v}_A$$

$$\bar{ab} = \bar{ob} - \bar{oa}$$

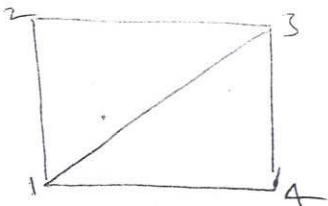
From above, we conclude that the relative velocity of point A with respect to B (v_{AB}) and the relative velocity of point B with respect to A (v_{BA}) are equal in magnitude but opposite in direction, i.e.,

$$v_{AB} = -v_{BA} \quad (\text{or}) \quad \bar{ba} = -\bar{ab}$$

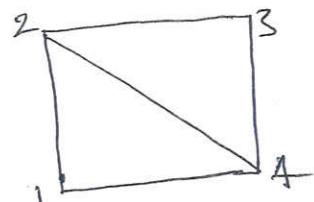
* Instantaneous centre for slider crank mechanism



$132 - 12, 23, 13$
 13
 $134 - 14, 34, 13$



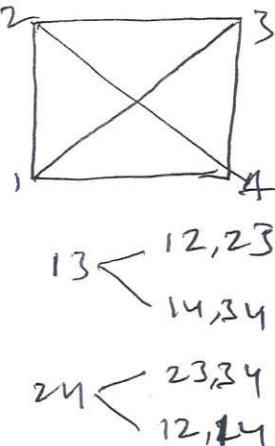
$241 - 12, 14, 24$
 24
 $243 - 23, 34, 24$



Locate the various I-centre as follows :-

1) Locate I-centre 12, 23 and 34 by inspection.

They are at the pivots joining the respective links. As the line of stroke of the slider is horizontal, the I-centre 1M lies vertically upwards or downwards at infinity.



2) I-centre 24 lies at intersection of lines joining the I-centre 12, 14 and 23, 34 by Kennedy's theorem. Joining of 12 and 14 means a vertical line through 12.

3) I-centre 13 lies at the intersection of lines joining the I-centres 12, 23 and 14, 34. Joining of 34 and 14 means a vertical line through 34.

I-centre 24 is a point common to links 2 and 4. Thus point 24 can be considered to be a point lying on and moving with either of the links 2 and 4. Thus, as a point on link 2, point 24 can be assumed to be rotating about 'O' with w2.

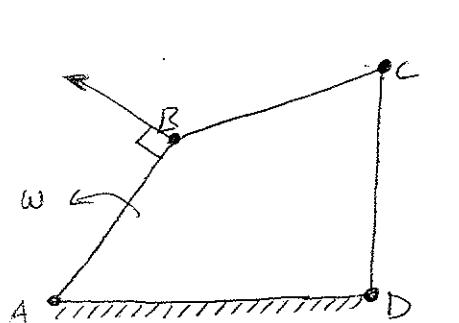
$$V_{24} = (12, 23) \omega_2 = V_{\text{Slider}}$$

$$V_{23} = (12, 23) \overset{\text{C.R.}}{\cancel{\omega_2}} = (13, 23) \omega_3$$

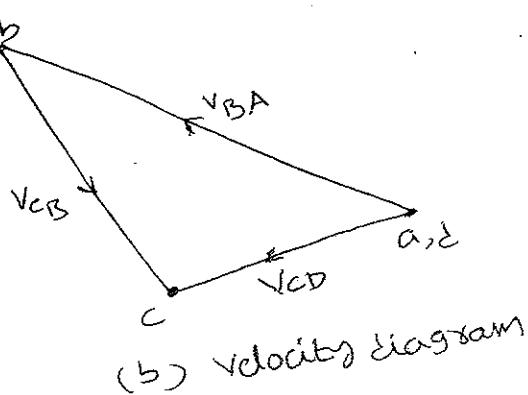
$$V_{34} = (13, 34) \omega_3 = V_{\text{Slider}}$$

B) Velocities in Four Bar chain :-

A four bar chain ABCD is shown in fig. The link AD is fixed and link AB is rotating anti-clockwise with a known angular velocity.



(a) Space diagram



(b) Velocity diagram

Let ω = Angular velocity of link AB

All the points lying on the link AB will have the same angular velocity i.e ω . Then linear velocity of B with respect to A (i.e., v_{BA}) is given by

$$\begin{aligned} v_{BA} &= \text{angular velocity of link AB} * \text{Length AB} \\ &= \omega * AB \end{aligned}$$

As the point A is fixed, hence the velocity of B with respect to A will be equal to velocity of B also i.e,

$$v_{BA} = v_B$$

This velocity will be perpendicular to link AB as shown in Fig(a).

The velocity diagram shown in Fig(b), is drawn as discussed below.

- (1) Take the fixed point a anywhere. Choose a suitable velocity scale and draw vector ab perpendicular to AB to represent the velocity of B with respect to A, i.e., V_{BA} .
- (2) The points A and D are fixed points. The fixed points of mechanism are coincident in the velocity diagrams. Therefore the point d coincides with the point a in the velocity diagram.
- (3) The velocity of c is with respect to point B and also with respect to point D. The velocity of c with respect to B is perpendicular to BC whereas the velocity of c with respect to D is \perp to CD.

From b draw vector bc \perp to BC to represent the velocity of c with respect to B i.e., V_{cb} . Also from 'a' or 'd' draw vector dc \perp to DC to represent the velocity of c with respect to D i.e., V_{cd} . The vectors bc and dc (or ac) intersect at 'c'. Then abc represents the velocity diagram for the given four bar chain.

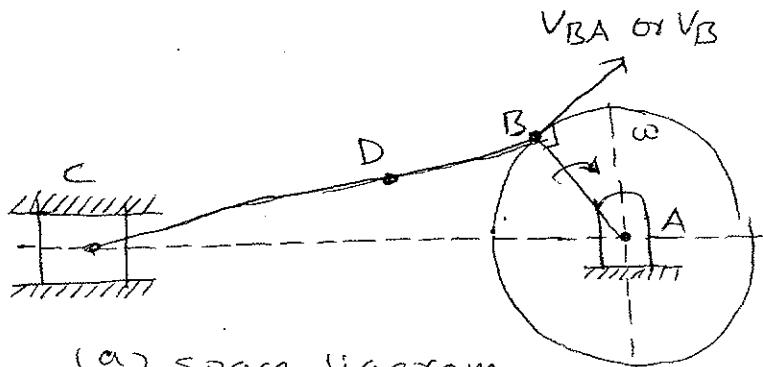
Now, we can determine the angular velocities of links BC and CD if required as:

$$\text{angular velocity of BC} = \frac{\text{vector } bc}{\text{length BC}}$$

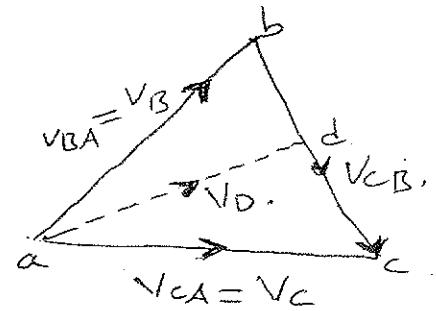
$$\text{and angular velocity of CD} = \frac{\text{vector } cd}{\text{length CD}}$$

④ VELOCITIES in Slider Crank mechanism :-

The Following Fig. Shows a slider crank mechanism, in which the crank AB is rotating clockwise with a given speed. The Slider 'C' is moving along the line CA. The connecting rod is represented by 'CB'.



(a) Space diagram



(b) Velocity diagram.

Let N = Speed of crank BA in r.p.m

$$\begin{aligned} \omega &= \text{angular velocity of crank BA} \\ &= \frac{2\pi N}{60} \end{aligned}$$

Then linear velocity of B with respect to A is given by

$$V_{BA} \text{ or } V_B = \omega * AB$$

This velocity will be perpendicular to AB as shown in Fig(a), The velocity diagram shown in Fig(b), is drawn as discussed below:

- ① Take the fixed point 'a' anywhere. choose a suitable velocity scale and draw vector ab perpendicular to AB to represent the velocity B with respect to A i.e., V_{BA} such that vector $ab = V_{BA}$.
- (2) The velocity of C is with respect to B and also with respect to A. The velocity of C with respect to B is bc to BC where - as the velocity of C with respect to A is in horizontal direction along CA. Hence From b draw vector bc perpendicular to BC to represent the velocity of C with respect to B i.e. V_{CB} .

But relative velocities to A, C can move horizontally, hence 'C' must lie somewhere on a horizontal line through A. Hence from point A draw a vector AC horizontally. The vector BC and AC intersect at 'C'. Then ABC represents the velocity diagram for the given slider crank mechanism.

3) The absolute velocity of any point 'D' on the connecting rod BC can be obtained by dividing vector BC such that,

$$\frac{bd}{bc} = \frac{BD}{BC}$$

$$bd = \frac{BD}{BC} * bc$$

As BD, BC and vector BC are known, hence bd can be calculated. Then location of point 'd' on vector BC is obtained. Now join point A to d. Then vector AD represents in magnitude and direction the absolute velocity of the point D.

If 'D' is the mid-point of connecting rod BC, then corresponding point 'd' will also be the mid-point of vector BC.

PROBLEMS

D In a Four bar chain ABCD, AD is fixed and is 15cm long. The crank AB is 4cm long and rotates at 120 r.p.m. clockwise, while the link CD (=8cm) oscillates about D. BC and AD are of equal length. Find the angular velocity of link CD when angle $BAD = 60^\circ$.

Sol:- Given:-

AD is Fixed and = 15cm, Crank AB = 4cm

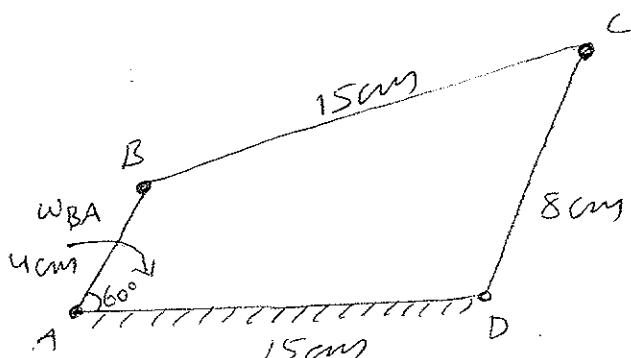
Speed of Crank AB, $\omega_{BA} = 120 \text{ r.p.m.}$

Link CD = 8cm, BC = AD = 15cm and Angle $BAD = 60^\circ$

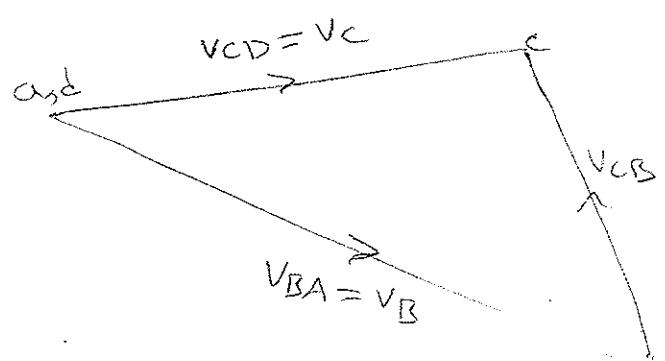
Draw the Space diagram, as shown in Fig (a) to a suitable scale.

Now Angular velocity of the link AB,

$$\omega_{BA} = \frac{2\pi N_{BA}}{60} = \frac{2\pi \times 120}{60} = 12.56 \text{ rad/sec.}$$



(a) space diagram.



(b) velocity diagram.

Velocity of B with respect to A (i.e., v_{BA}) or simply velocity of B as A is fixed (i.e., v_B) is given by

$$v_{BA} \text{ (or } v_B) = \omega_{BA} * \text{Length AB}$$

$$v_{BA} \text{ or } v_B = 12.56 * 4 = 50.24 \text{ cm/sec.}$$

On measurement from velocity triangle, velocity of C with respect to D is given by

$$v_{CD} = \text{vector dc (or ac)} = 38 \text{ cm/sec}$$

Angular velocity of link CD is given by

$$\omega_{CD} = \frac{v_{CD}}{\text{Length CD}} = \frac{15+15+8}{8} = \frac{38}{8} = 4.75 \text{ rad/sec}$$

----- * -----

2)

The crank of a slider crank mechanism is 15cm and the connecting rod is 60cm long. The cranks makes 300 rpm in the clockwise direction. When it has turned 45° from the inner dead centre position, determine

(i) velocity of slider C,

(ii) Angular velocity of connecting rod and

(iii) linear velocity of the mid-point of the connecting rod.

Sol:- Given:-

Crank Length, AB = 15cm

connecting rod length, BC = 60cm

Crank Speed, $N = 300 \text{ rpm}$

Angle turned by crank I.D.C = 45° .

Unit 2 - Pg-17/31

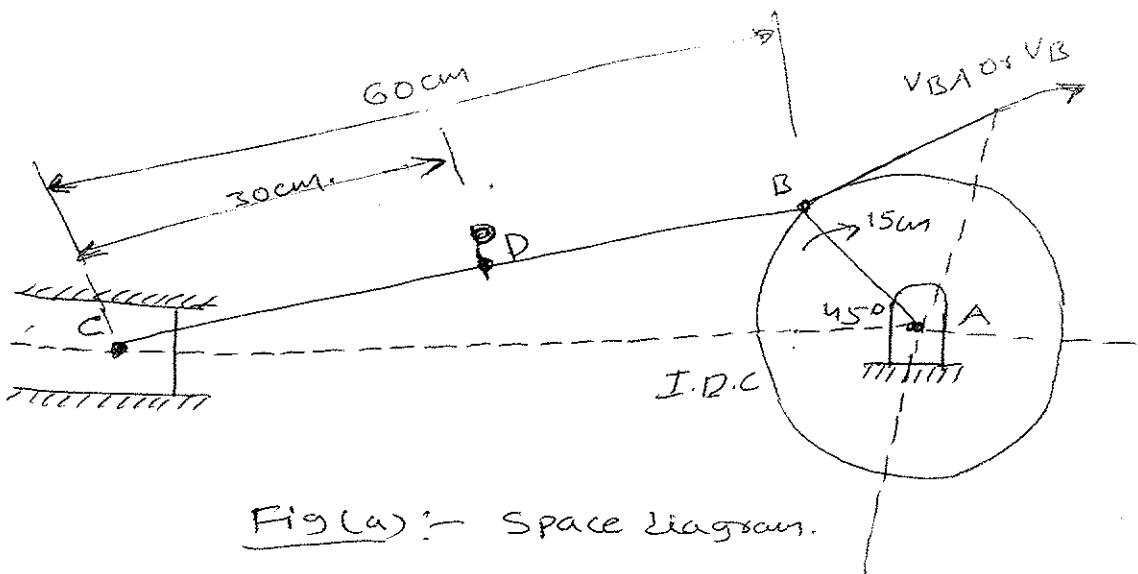


Fig (a) :- Space diagram.

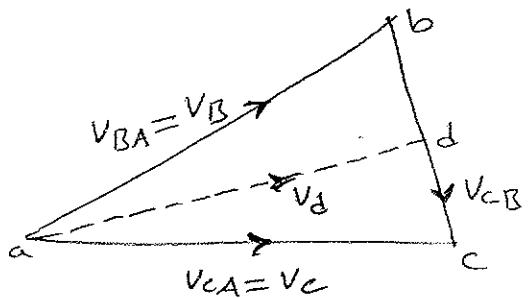


Fig (b) :- Velocity diagram -

Now angular velocity of crank is given by,

$$\omega = \frac{2\pi * N}{60} = \frac{2\pi * 300}{60} = 31.42 \text{ rad/sec.}$$

∴ Linear velocity of B with respect to A (or velocity of B) is given by,

$$v_{BA} = v_B = \omega * AB = 31.42 * 15$$

$$v_{BA} = v_B = 471.3 \text{ cm/sec.}$$

$$\text{The vector } ab = v_{BA} = v_B = 471.3 \text{ cm/sec.}$$

By measurement, we find velocity of 'C' with respect to 'B'.

$$V_{CB} = \text{vector } BC = 340 \text{ cm/sec}$$

$$V_{CAB} = V_{BA} - BC + AC$$

$$V_{CAB} = 471 - (60 + 60 + 15)$$

$$V_{CAB} = 336.3 \approx 340 \text{ cm}$$

and velocity of C with respect to A or simply velocity
of C,

$$V_{CA} \text{ or } V_C = \text{vector } AC = 60 + 340 = 400 \text{ cm/sec}$$

(i) velocity of Mid of C = $V_C = 400 \text{ cm/sec}$

(ii) Angular velocity of connecting rod CB is given by

$$\omega_{CB} = \frac{V_{CB}}{\text{Length } BC} = \frac{340}{60}$$

$$\boxed{\omega_{CB} = 5.67 \text{ rad/sec}}$$

iii linear velocity of the mid-point of the connecting
rod

by measurement, we find that

$$V_D = \text{vector } AD = 410 \text{ cm/sec}$$

 *

45-

$$V_D = V_{BA} - (\omega + \text{mid-point of connecting rod})$$

$$\frac{31}{61}$$

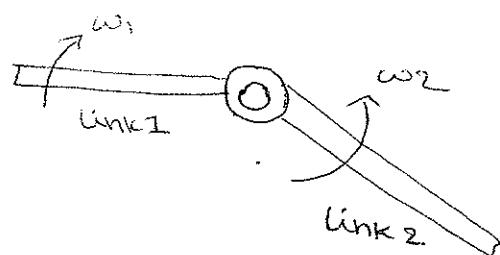
$$= 471.3 - (31.42 + 30)$$

$$\frac{471}{60} \\ 410.$$

400

unit 2, Pg-19(3)

Rubbing velocity at a Pin-Joint



The two links 1 and 2 are connected by means of a pin-joint as shown in Fig. above

Let

ω_1 = Angular velocity of Link 1,

ω_2 = Angular velocity of Link 2, and

r = radius of the pin at the joint.

The rubbing velocity is defined as the algebraic difference between the angular velocities of the two links which are connected by pin-joints, multiplied by the radius of the pin.

Hence the rubbing velocity at the pin joint, when the two connected links move in opposite direction is given by,

$$\text{Rubbing velocity} = r(\omega_1 + \omega_2)$$

But if the two connected links move in the same direction, Then

$$\text{Rubbing velocity} = r(\omega_1 - \omega_2)$$

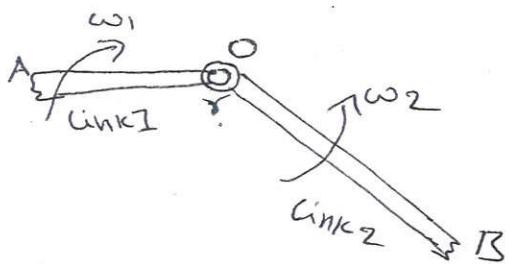
If a pin connects one sliding member and the other turning (For ex. gudgeon pin of a connecting rod) the angular velocity of the member is zero and hence the velocity of rubbing will be given by,

$$\text{Rubbing velocity} = r\omega$$

where ω = angular velocity of the sliding member
 r = radius of the pin.

Two links OA and OB are connected by a pin joint at 'O'. The link OA turns with angular velocity 10 rad/sec in clockwise direction and the link OB turns with angular velocity 6 rad/sec, in anticlockwise direction. If the radius of the pin at 'O' is 8 mm, then what is the rubbing velocity, at the Joint?

Sol:-



~~Given~~ given that $\omega_1 = 10 \text{ rad/sec}$, clockwise
 $\omega_2 = 6 \text{ rad/sec}$, anti-clockwise
 $r = 8 \times 10^3 \text{ m.}$

when the two connected links move in opposite directions,

$$\begin{aligned}\text{The Rubbing velocity } (v) &= (\omega_1 + \omega_2)r \\ &= (10 + 6) \times 8 \times 10^3 \\ &= 16 \times 8 \times 10^3 = 0.128 \text{ mm/sec.}\end{aligned}$$

Note:- when the two connected links move in the same direction.

$$\begin{aligned}\text{The Rubbing velocity } (v) &= (\omega_1 - \omega_2)r \\ &= (10 - 6) \times 8 \times 10^3 \\ &= 4 \times 8 \times 10^3 \\ &= 32 \times 10^3 \\ v &= 0.032 \text{ mm/sec.}\end{aligned}$$

1) Coriolis acceleration component :-

It is seen that the acceleration of a moving point relative to a fixed body (fixed co-ordinate system) may have 2 components of acceleration, the centripetal & the tangential. However in some cases, there may have its motion relative to a moving body (moving coordinate system, for example, motion of slider on a rotating link). The following analysis is made to investigate the acceleration at that point.

Let a link AR rotate about a fixed point A' on it. P is a point on a slider on the link.

At any given instant.

Let ω = angular velocity of the link

α = angular acceleration of the link

v = linear velocity of the slider on link

f = linear acceleration of the " " " slider.

r = radial distance of point 'P' on slider

In a short interval of time δt , let $\delta\theta$ be the angular displacement of the link $\frac{\delta\theta}{\delta t}$, δr the radial displacement of the slider in the outer direction after the short interval time δt , let

$$\omega' = \omega + \alpha \delta t = \text{angular velocity of link}$$

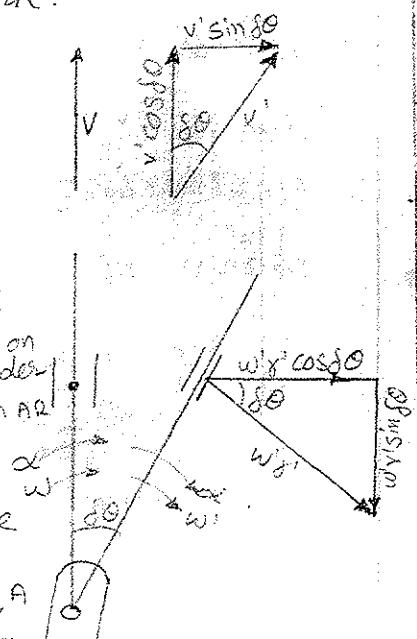
$$v' = v + f \cdot \delta t = \text{linear velocity of the slider on the link}$$

$$r' = r - \delta r = \text{radial distance of the slider}$$

∴ Acceleration of 'P' parallel to 'AR' :-

$$\text{Initial velocity of } P \text{ along } AR = v = v_{pq}$$

$$\text{Final velocity of } P \text{ along } AR = v' \cos \delta\theta - \omega' \delta r \sin \delta\theta$$



change of velocity along AR = $(v' \cos \delta\theta - w' \delta\theta \sin \delta\theta) - v$

Acceleration of 'P' along AR = $\frac{(v + f\delta t) \cos \delta\theta - (w + \alpha \delta t)(\delta\theta + \delta\delta) \sin \delta\theta - v}{\delta t}$
in the limit, as $\delta t \rightarrow 0$

$$\cos \delta\theta \rightarrow 1 \quad \delta\theta \rightarrow \delta\theta$$

$$\begin{aligned} \text{acceleration of 'P' along 'AR'} &= f - w\delta\theta \frac{d\theta}{dt} \\ &= f - w\delta\theta \cdot \omega = f - w^2\delta\theta \end{aligned}$$

= acc. of slider - centripetal acc

This is the acceleration of 'P' along AR in the radially outward direction. f will be (-ve) if the slider has deceleration while moving in the outward direction or has acceleration while moving in the inward direction.

ii Acceleration of 'P' \perp to AR :-

$$\text{Initial velocity of } P \perp \text{ to AR} = w\delta\theta$$

$$\text{Final velocity of } P \perp \text{ to AR} = v' \sin \delta\theta + w' \delta\theta \cos \delta\theta$$

$$\text{change of velocity } \perp \text{ to AR} = (v' \sin \delta\theta + w' \delta\theta \cos \delta\theta) - w\delta\theta$$

acceleration of $P \perp$ to AR = $\frac{(v + f\delta t) \sin \delta\theta - (w + \alpha \delta t)(\delta\theta + \delta\delta) \cos \delta\theta - w\delta\theta}{\delta t}$
in the limit, as $\delta t \rightarrow 0$

$$\cos \delta\theta \rightarrow 1 \quad \delta\theta \rightarrow \delta\theta$$

$$\begin{aligned} \text{acceleration of } P \perp \text{ to AR} &= v \frac{d\theta}{dt} + w \frac{dr}{dt} + r\alpha \\ &= vw + wr + r\alpha \\ &= 2vw + r\alpha \\ &= 2vw + \text{tangential acceleration} \end{aligned}$$

This is the acceleration of 'P' \perp to 'AR'. The component $2vw$ is known as the coriolis acceleration component. It is (+ve) if both w & v are either +ve or -ve.

→ Thus, the coriolis component is +ve if the

- link AR rotates clock-wise & the slider moves radially outwards

- link rotates counter-clockwise & the slider moves radially inwards.

Otherwise, the coriolis component will be negative.

→ These observations can be summarised into the following rule.

→ The dissection of the coriolis acceleration component is obtained by rotating the radial velocity vector 'v' through 90° in the direction of rotation of the link.

→ Let 'Q' be a point on the link AR immediately beneath the Point 'P' at the instant. Then acc. of P = acceleration of P|| to AR + acc. of P ⊥ to AR.

$$f_{pa} = (f - \omega^2 r) + (2\omega v + \gamma \alpha)$$

$$= f + (\gamma \alpha - \omega^2 r) + 2\omega v$$

= acc. of P relative to Q + acc. of Q relative to A + coriolis acceleration component.

$$= f'_{pq} + f_{qa} + f^{cr}$$

In the above eqn, f'_{pq} is the acc. which an observer stationed on link AR would observe for the slider. Remember that coriolis component exists only if there are two coincident points which have

- linear relative velocity of sliding &
- angular motion about fixed finite centres of rotation.

Sometimes for the sake of simplicity, it is convenient to associate the coriolis acceleration component f^{cr} with f'_{pq} & writing the eqn in this form.

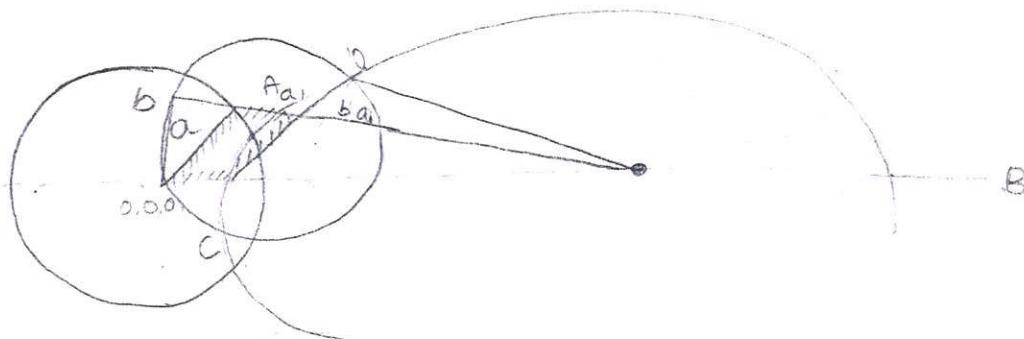
where, $f_{pa} = f_{pq} + f_{qa}$

$$f_{pq} = f'_{pq} + f^{cr}$$

This makes solving problems quite easy

2) Klein's Construction :-

In Klein's construction, the velocity & the acceleration diagrams are made on the configuration diagram itself. The line that represents the crank in the configuration diagram also represents the velocity & the acceleration of its moving end in the velocity & the acceleration diagrams, respectively. For a slider-crank mechanism, the procedure to make the Klein's construction is described below.



Slider-Crank mechanism :-

In fig. 3.25, OAB represents the configuration of a slider-crank mechanism. Its velocity & acceleration diagrams are as shown in fig. 34(b) & (c). Let 's' be the length of the crank OA.

Velocity diagram :-

For velocity diagram, let 's' represent V_{AO} , to some scale. Then for the velocity diagram, length $OA = \omega s = CA$. From this, the scale for the velocity diagram is known. Produce BA & draw a line \perp to OB through 'O'. The intersection of the two lines locates the point 'b'. The fig. OAB is the velocity diagram which is similar to the velocity diagram of fig. 3.4(b) rotated through 90° in a direction opposite to that of the crank.

Acceleration diagram :-

For acceleration diagram, let 's' represent for θ

$$O_A = \omega^2 s = OA$$

This provides the scale for the acceleration diagram make the following construction.

- Draw a circle with ab as the radius & a as the centre
- Draw another circle with AB as diameter.
- Join the points of intersections C & D of the two circles, let it meet OB at b, & AB at E.
- Then o,a,b,a, is the required acceleration diagram which is similar to the acceleration diagram of fig. 3.4(c) rotated through 180° .

The proof is as follows

Join AC & BC

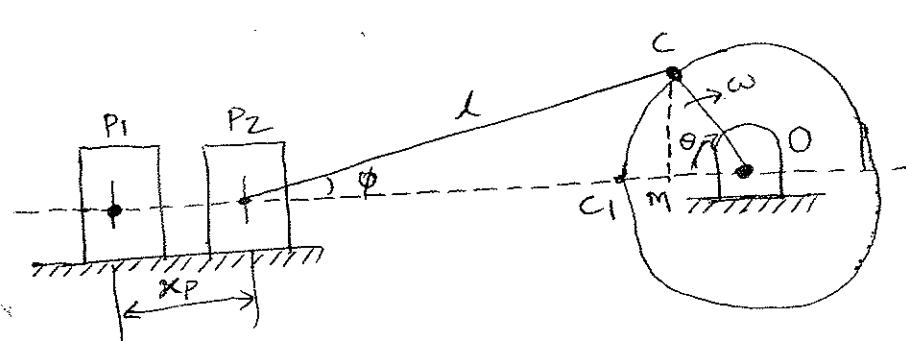
AEC & ABC are two right-angled triangles in which the angle CAB is common. Therefore, the triangles are similar

$$\frac{AE}{AC} = \frac{AC}{AB} \text{ (or)} AE = \frac{(AC)^2}{AB} \text{ (or)} a_{ba} = \frac{(ab)^2}{AB} = f_{ba}$$

Thus, this acceleration diagram has all the sides, \parallel to that of acceleration diagram of fig 3.4(c) & also the has 2 sides, a_A & a_{ba} representing the corresponding magnitudes of the acceleration. Thus, the two diagrams are similar.

D Analytical determination of velocity and acceleration of the piston :-

consider the reciprocating engine mechanism, Figure shows crank mechanism in a position making an angle θ with the inner Dead centre position OC_1 . Let the crank be of length γ and let it rotate uniformly in clockwise direction with angular speed ω rad/sec. Let $\lambda = (n)\gamma$ be the length of the connecting rod. Let ϕ be the angle.



when the crank is at the inner Dead centre position OC_1 , the connecting rod C_1P_1 lies along the line of stroke and for this position.

$$OP_1 = (\lambda + r)$$

At the moment when the crank rotates through ' θ ' in clockwise direction, the distance of piston from crank shaft centre is given by

$$OP_2 = (\lambda \cos\phi + r \cos\theta)$$

thus, displacement x of the piston 'P' from inner Dead centre position is given by:

$$\begin{aligned}
 \vec{r}_P &= \vec{r}_1 \vec{r}_2 \\
 &= \vec{OP_1} - \vec{OP_2} \\
 &= [\vec{OC} + \vec{CP_1}] - [\vec{OM} + \vec{MP_2}] \\
 &= (r + d) - (r \cos\theta + h \cos\phi) \\
 &= (r + hr) - \left[r \cos\theta + h \cdot r \sqrt{1 - \frac{\sin^2\theta}{h^2}} \right]
 \end{aligned}$$

$\lambda_P =$

$$= r + hr - r \cos\theta - hr \sqrt{1 - \frac{\sin^2\theta}{h^2}}$$

$$= r(1+h) - r \underbrace{\left[\cos\theta + \sqrt{h^2 - \sin^2\theta} \right]}_{\lambda}$$

$$\begin{aligned}
 \therefore \lambda &= hr \\
 cm &= r \sin\theta = \cancel{rsin\theta} \\
 \cancel{sin\theta} &= \cancel{r sin\theta} \\
 \cos\theta &= \sin\theta/h
 \end{aligned}$$

$$\lambda_P = r(1 - \cos\theta) + r(h - \sqrt{h^2 - \sin^2\theta})$$

$$\sin\theta = \frac{rsin\theta}{r}$$

$$\sin\theta = \frac{\sin\theta}{h}$$

$$\cos\theta = \sqrt{1 - \frac{\sin^2\theta}{h^2}}$$

Velocity Equation :-

$$V = \frac{dr}{dt}$$

$$V = \frac{dr}{d\theta} * \frac{d\theta}{dt}$$

$$V = \omega \cdot \frac{dr}{d\theta}$$

$$V = \omega \frac{1}{d\theta} \left\{ r(1 - \cos\theta) + r(h - \sqrt{h^2 - \sin^2\theta}) \right\}$$

$$V = \omega \frac{1}{d\theta} \left\{ (1 - \cos\theta) + (h - \sqrt{h^2 - \sin^2\theta}) \right\}$$

$$V = \gamma \omega \left\{ \theta - (-\sin \theta) + \left(\theta - \frac{1}{2} (\dot{\theta}^2 - \sin^2 \theta) \right)^{1/2} \right\}$$

$$V = \gamma \omega \left\{ \sin \theta + \frac{\sin 2\theta}{2\sqrt{\dot{\theta}^2 - \sin^2 \theta}} \right\}$$

$$\therefore V = \gamma \omega \left\{ \sin \theta + \frac{\sin 2\theta}{2\sqrt{\dot{\theta}^2 - \sin^2 \theta}} \right\} \quad \left[\because \sin 2\theta = 2 \sin \theta \cos \theta \right]$$

Acceleration :-

$$V_{\text{approx}} = \gamma \omega \left\{ \sin \theta + \frac{\sin 2\theta}{2h} \right\} \quad \left[\because \text{neglecting higher order term} \right]$$

$$\Rightarrow a = \frac{dv}{dt} = \frac{dv}{d\theta} \cdot \frac{d\theta}{dt}$$

$$a = \omega \frac{d}{d\theta} \left\{ \gamma \omega \left[\sin \theta + \frac{\sin 2\theta}{2h} \right] \right\}$$

$$a = \gamma \omega^2 \frac{d}{d\theta} \left\{ \sin \theta + \frac{\sin 2\theta}{2h} \right\}$$

$$a = \gamma \omega^2 \left\{ \cos \theta + \frac{\cos 2\theta \cdot 2}{2h} \right\}$$

$$\therefore a = \gamma \omega^2 \left\{ \cos \theta + \frac{\cos 2\theta}{h} \right\}$$

* Velocity and acceleration Analysis of connecting rod:

Also, from $\sin\phi = \frac{x}{n} \sin\theta$

Differentiating on both sides with respect to time, we have

$$(\cos\phi) \frac{d\phi}{dt} = \frac{x}{n} \cos\theta \frac{d\theta}{dt}$$

$$\frac{d\phi}{dt} = \omega_s, \frac{d\theta}{dt} = \omega$$

$$(\cos\phi) \omega_s = \frac{\cos\theta}{n} \frac{d\theta}{dt}$$

$$\omega_s = \frac{\omega}{n} \frac{\cos\theta}{\cos\phi}$$

(cos)

$$\omega_{CR} = \frac{\omega}{n} \frac{\cos\theta}{\cos\phi}$$

$$\therefore \cos\phi = \sqrt{1 - \frac{\sin^2\theta}{n^2}}$$

$$\cos\phi = \frac{\sqrt{n^2 - \sin^2\theta}}{n}$$

$$\omega_{CR} = \frac{\omega \cdot \cos\theta}{\sqrt{n^2 - \sin^2\theta}}$$

$$\omega_{CR} = \frac{\omega \cdot \cos\theta}{\sqrt{n^2 - \sin^2\theta}}$$

∴ The angular velocity of connecting rod is

$$\boxed{\omega_{CR} = \frac{\omega \cdot \cos\theta}{\sqrt{n^2 - \sin^2\theta}}}$$

Now, acceleration Analysis of connecting rod is.

$$\alpha_{CR} = \frac{d}{dt} \omega_{CR} = \frac{d}{dt} \left[\frac{\omega \cos\theta}{\sqrt{n^2 - \sin^2\theta}} \right]$$

$$\alpha_{CR} = \omega \frac{d}{dt} \left[\cos\theta \cdot (n^2 - \sin^2\theta)^{-1/2} \right]$$

$$\alpha_{cr} = \omega^2 \left[\cos\theta - \frac{1}{2} (n^2 \sin^2\theta)^{-3/2} \cdot \left(-2 \sin\theta \cdot \cos\theta \right) + (n^2 \sin^2\theta)^{1/2} \cdot \left(-\sin\theta \right) \right] \frac{d\theta}{dt}$$

$$\alpha_{cr} = \omega^2 \left\{ \frac{\sin 2\theta \cdot \cos \theta}{2(n^2 \sin^2\theta)^{3/2}} - \frac{\sin \theta}{(n^2 \sin^2\theta)^{1/2}} \right\}$$

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